A note on Units

You may have taken another theory course at some point, where "theory units" were used, that appeared to set all the fundamental constants equal to one. However, since the fine structure constant is always $\alpha = e^2/\hbar c = 1/137$, you can't simultaneously set e, $\hbar$, and c all equal to one. In the Lorentz-Heaviside units used in relativistic quantum field theories, $c = \hbar = 1$, and the electron charge is therefore $e = \sqrt{\alpha}$. In nonrelativistic quantum mechanics, as we are primarily interested in here, c is not the natural unit of velocity, but e is certainly a natural unit of electric charge, and so in atomic units $e = \hbar = m_e = 1$, where m_e is the mass of the electron. Then $c = 1/\alpha = 137$ is a big number in atomic units. This is appropriate, since the electron's "natural" speed, in the ground state of hydrogen, is 137 times slower than c.

Table I summarizes conversion factors between atomic units, cgs "Gaussian" units, and MKSA units. In general, atomic units are defined by the velocities, forces, etc., experienced by the electron in the ground state of hydrogen. However, there is some ambiguity in defining the atomic unit of magnetic field, so we have included two: the "standard" unit represents a unit field as one that imparts a unit Lorentz force to a unit charge moving at unit velocity; while the "Gaussian" unit asserts that the electric and magnetic fields in a plane wave have the same magnitude. Using the Gaussian definition therefore retains the form of Maxwell's equations in Gaussian units. The Gaussian to MKSA conversion for various equations of electromagnetism are given in Table II.

In real-life spectroscopy, there are other units used for various quantities. Table III summarizes some of these, for your viewing pleasure.

Table I A miscellany of physical quantities, in three systems of units.

Physical Quantity	**Atomic Units**	**CGS "Gaussian"**	**MKSA**
Electron charge e	1	4.8×10^{-10} esu	1.6×10^{-19} Coulomb
Electron mass m_e	1	9.1×10^{-28} g	9.1×10^{-31} kg
Reduced Planck constant $\hbar$	1	1.05×10^{-27} ergs	1.05×10^{-34} Joules
Fine structure constant α	$e^2/\hbar c = 1/137$	$1/137$	$e^2/(4\pi\epsilon_0\hbar c) = 1/137$
Speed of light c	$1/\alpha = 137$	3.0×10^{10} cm/sec	3.0×10^{8} m/sec
Bohr radius a_0	$\hbar^2/(m_e e^2) \equiv 1$ Bohr	5.3×10^{-9} cm	$4\pi\epsilon_0\hbar^2/(m_e e^2) = 5.3 \times 10^{-11}$ m
Potential energy of an electron 1 a_0 from a proton (=2 Ry)	$e^2/a_0 \equiv 1$ Hartree	4.4×10^{-11} ergs	$e^2/(4\pi\epsilon_0 a_0) = 4.4 \times 10^{-18}$ Joules
Angular frequency of classical electron in first Bohr orbit	$\frac{\text{Hartree}}{\hbar} = 1$	4.2×10^{16} rad/sec	4.2×10^{16} rad/sec
Atomic unit of time	$\hbar^3/(m_e e^4) = 1$	2.4×10^{-17} sec	2.4×10^{-17} sec
Electric potential on electron a_0 from proton	$e/a_0 = 1$	9×10^{-2} statvolts	$e/(4\pi\epsilon_0 a_0) = 27$ Volts
Electric field on electron a_0 from proton	$e/a_0^2 = 1$	1.7×10^{7} statvolts/cm	$e/(4\pi\epsilon_0 a_0^2) = 5.2 \times 10^{11}$ V/m
Electric dipole moment of electron a_0 from proton	$ea_0 = 1$	2.5×10^{-18} statvolt-cm	$ea_0 = 8.5 \times 10^{-30}$ Coulomb-meters
"Standard" atomic unit of magnetic field	1	2.5×10^{9} gauss	2.5×10^{5} Tesla
"Gaussian" atomic unit of magnetic field	$\alpha\times$ "standard"	1.7×10^{7} gauss	1.7×10^{3} Tesla
Bohr magneton μ_B	$e\hbar/(2m_e c) = \alpha/2 = \frac{1}{274}$	9.3×10^{-21} ergs/gauss	$e\hbar/(2m_e) = 9.2 \times 10^{-24}$ Joules/Tesla

Table II Sundry equations of electromagnetism, in both Gaussian and MKSA units.

Relation	**Gaussian**	**MKSA**
Maxwell's equations	$\vec{\nabla}\cdot\vec{B}=0$ $\vec{\nabla}\cdot\vec{E}=4\pi\rho$ $\vec{\nabla}\times\vec{E}+\frac{1}{c}\frac{\partial\vec{B}}{\partial t}=0$ $\vec{\nabla}\times\vec{B}-\frac{1}{c}\frac{\partial\vec{E}}{\partial t}=\frac{4\pi}{c}\vec{J}$	$\vec{\nabla}\cdot\vec{B}=0$ $\vec{\nabla}\cdot\vec{E}=\rho$ $\vec{\nabla}\times\vec{E}+\frac{\partial\vec{B}}{\partial t}=0$ $\vec{\nabla}\times\vec{B}-\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}=\mu_0\vec{J}$
Vector potential	$\vec{E}=-\vec{\nabla}\phi-\frac{1}{c}\frac{\partial\vec{A}}{\partial t}$ $\vec{B}=\vec{\nabla}\times\vec{A}$	$\vec{E}=-\vec{\nabla}\phi-\frac{1}{c}\frac{\partial\vec{A}}{\partial t}$ $\vec{B}=\vec{\nabla}\times\vec{A}$
Force law	$\vec{F}=q\left(\vec{E}+\frac{1}{c}\vec{v}\times\vec{B}\right)$	$\vec{F}=q\left(\vec{E}+\vec{v}\times\vec{B}\right)$
Potential due to point charge	$\phi=\frac{q}{r}$	$\phi=\frac{1}{4\pi\epsilon_0}\frac{q}{r}$
Energy of field	$U=\frac{1}{8\pi}\left(E^2+B^2\right)$	$U=\frac{1}{2}\left(\epsilon_0E^2+\frac{1}{\mu_0}B^2\right)$
Energy of dipole in a field	$U_E=-\vec{p}\cdot\vec{E}$ $U_M=-\vec{\mu}\cdot\vec{B}$	$U_E=-\vec{p}\cdot\vec{E}$ $U_M=-\vec{\mu}\cdot\vec{B}$
Field on axis of a loop of radius a	$\frac{2\pi I}{ac}\left(1+z^2/a^2\right)^{-3/2}$	$\frac{\mu_0 I}{2a}\left(1+z^2/a^2\right)^{-3/2}$

Table III Other useful units.

1 wave number = 1 cm^{-1}	30 GHz	1.4 Kelvin	2.0×10^{-16} ergs	
1 electron-Volt = 1 eV	2.4×10^{14} Hz	1/300 electron-statvolt	1.2×10^{4} Kelvin	1.6×10^{-19} J
1 W/cm^2	2.8×10^{-17} atomic units	10^{4} ergs/(cm^2-sec)	26 V/cm peak electric field	
1 Debye (electric dipole moment)	10^{-18} esu-cm	1/2.54 atomic units	3.4×10^{-30} Coulomb-m	
1 cm		1/2.54 inches		